A Chart of the Northern Hemisphere on an equal-surface projection, showing all the Stars in Argelander's Series of forty full-sheet Charts, 324,198 *in all, with a Key-Map on the same projection.* By R. A. Proctor, B.A., F.R.A.S. Manchester: Photographed and Published by A. Brothers, 1871.

In this Chart, within a circle 11 inches in diameter, are 324,198 minute dots, each representing a star, copied in with careful attention, both to position and size, from Argelander's series of star-charts: the chart thus showing the laws according to which the stars down to the 9–10th magnitude are distributed over the northern hemisphere.

The Chart being intended specially to show where the stars are richly and where sparsely strewn, the projection is such that equal spaces on the celestial vault are represented by equal spaces on the chart. The chart in the original scale formed a circle two feet in diameter: this was divided by meridians and parallels (radii and concentric circles) into 26,400 spaces, answering to corresponding spaces in Argelander's series: and into these spaces the stars were copied: the work of charting occupied almost exactly 400 hours, giving an average of 4½ seconds for each star. By means of Photography the work has been reproduced more satisfactorily than it could have been done by any engraving, however skilfully drawn: and the author notices that in this case, as in his Star-atlas, he has been most fortunate in obtaining the assistance of a fellow-astronomer (to whom such work is a labour of love).

Memoir by Prof. S. Newcomb on the Lunar Theory.

There is contained in the *Comptes Rendus* for 3 April, 1871, an account, by Prof. S. Newcomb, of a memoir, "Théorie des perturbations de la Lune qui sont dues à l'action des Planètes," presented by him to the Academy. The author seeks to avoid certain difficulties by regarding the perturbing force of the Sun as a principal force, and by proposing the problem as follows:

"*Assuming the solution of the problem of three bodies, to find the perturbations produced by the action of a fourth body, by the method of the variation of the arbitrary constants, making use of Lagrange's general formulæ.*"

Let m_1, m_2, m_3, m_4 be the masses of the Sun, Earth, Moon and Planet; ϱ_1, ϱ_2, ϱ_3, the distances of the planet from the other three bodies; a_1, a_2, ... a_{18}, the arbitrary constants of the motion of the three bodies.

The disturbing function is

$$R = m_4\left(\frac{m_1}{\varrho_1} + \frac{m_2}{\varrho_2} + \frac{m_3}{\varrho_3}\right).$$

R being expressed as a function of the arbitrary constants and of t; the variations are given by 18 differential equations of the form

$$(a_i, a_1)\frac{d a_1}{d t} + (a_i, a_2)\frac{d a_2}{d t} \dots + (a_i, a_{18})\frac{d a_{18}}{d t} = \frac{d\,\mathrm{R}}{d a_i}$$

where, as usual,

$$(a_i, a_j) = \Sigma m \left\{ \frac{\delta(x, x')}{\delta(a_i, a_j)} + \frac{y}{\delta(a_i, a_j)} + \frac{\delta(z, z')}{\delta(a_i, a\ \ )} \right\}.$$

The number of combinations (a_i, a_j) is $= 153$: their direct calculation would be impracticable from its length, but it may be greatly simplified.

First, separating the six constants which determine the position of the centre of gravity of the three bodies from the other twelve, every combination (a, b) of one of the six constants with one of the twelve vanishes identically: and the combinations *inter se* of the six constants give simply the principle of the conservation of the centre of gravity: there remain to be considered only the 66 combinations of the 12 constants *inter se.*

The author takes—

a, e, γ, the constants usually spoken of as the mean distance of the Moon from the Earth, the excentricity of the orbit, and the inclination.

a', e', γ', the like constants for the motion of the common C. G. of the Earth and Moon about the Sun, γ' being the inclination of the ecliptic to the plane of X Y.

ε, π, θ, ε', π', θ', the mean longitude, longitude of perigee, and longitude of node for the Moon, and corresponding elements for the Sun or any six independent linear functions of these elements.

x, y, z, the co-ordinates of the Moon in reference to the Earth.

X, Y, Z, the co-ordinates of the Sun in reference to the common C. G. of the Moon and Earth.

The values of x, y, z, X, Y, Z, may be presented in the forms

$$x = \Sigma k \cos(i\,\varepsilon + i^{\mathrm{i}}\,\pi + i^{\mathrm{ii}}\,\theta + i^{\mathrm{iii}}\,\varepsilon' + i^{\mathrm{iv}}\,\pi' + i^{\mathrm{v}}\,\theta')$$
$$y = \Sigma k \sin(i\,\varepsilon + i^{\mathrm{i}}\,\pi + i^{\mathrm{ii}}\,\theta + i^{\mathrm{iii}}\,\varepsilon' + i^{\mathrm{iv}}\,\pi' + i^{\mathrm{v}}\,\theta')$$
$$z = \Sigma k' \sin(j\,\varepsilon + j^{\mathrm{i}}\,\pi + j^{\mathrm{ii}}\,\theta + j^{\mathrm{iii}}\,\varepsilon' + j^{\mathrm{iv}}\,\pi' + j^{\mathrm{v}}\,\theta')$$

X, Y, Z, being the like functions with K, K$'$, for k, k' (and k, k', K, K$'$, being functions of a, e, γ, a', e', γ').

It is shown that of the 66 terms (a_i, a_j) there are 30 which vanish, while the remaining 36 are the derivatives of 6 functions of a, e, γ, a', e', γ', with respect to these same constants: these 6 functions are represented by k_ε, k_π, k_θ, $k_{\varepsilon'}$, $k_{\pi'}$, $k_{\theta'}$, and are formed as follows:—

1°. Each of the angles $i\,\varepsilon + i'\,\pi + \dots, j\,\varepsilon + j'\,\pi' + \dots$, being of the form $\mathrm{A} + b\,t$ where b is a function of a, e, γ, a', e', γ', we form for each term of each of the co-ordinates [x, y, z]* the product $\frac{m_2\,m_3}{m_2 + m_3}\,b\,k^2$, and for [each term of]* each of the co-ordinates

* I have added these insertions in [ ], which seem to me to be the meaning. —Ed.

X Y, Z, the product $\frac{m_1(m_2+m_3)}{m_1+m_2+m_3}$ B k^2. These products are called h_1, h_2, h_3, H_1, H_2, H_3: and the different values of h_1, h_2, are identical as well as those of H_1, H_2.

2°. Multiply each h by the corresponding coefficient of ε, and take the half sum of these products as well for x as for each of the other five co-ordinates of the Sun and Moon. This half-sum is called k_ε; and the like for k_π, k_θ, $k_{\varepsilon'}, k_{\pi'}, k_{\theta'}$. Then

$$(a, \varepsilon) = -\frac{d k_\varepsilon}{d a}, \ (e, \varepsilon) = -\frac{d k_\varepsilon}{d e} \ \ldots \ (\gamma', \varepsilon) = \frac{d k_\varepsilon}{d \gamma'}$$

$$(a, \pi) = -\frac{d k_\pi}{d a}, \ (e, \pi) = -\frac{d k_\pi}{d e} \ \ldots \ (\gamma', \pi) = \frac{d k_\pi}{d \gamma'}$$

[that is, 36 equations $(a, b) = \mp \frac{d k_b}{d a}$, if a is any one of the constants $a, e, \gamma, a', e', \gamma'$, and b any one of the constants $\varepsilon, \pi, \theta, \varepsilon', \pi', \theta'$, and the sign being $-$ when the (a, b) are both unaccented or both accented, but $+$ if one is accented and the other unaccented.—Ed.]

The equations for the variations thus are

$$\frac{d k_\varepsilon}{d a}\frac{d a}{d t} + \frac{d k_\varepsilon}{d e}\frac{d e}{d t} \ldots + \frac{d k_\varepsilon}{d \gamma'}\frac{d \gamma'}{d t} = \frac{d \mathrm{R}}{d t}$$

$$\frac{d k_\pi}{d a}\frac{d a}{d t} + \frac{d k_\pi}{d e}\frac{d e}{d t} \ldots + \frac{d k_\pi}{d \gamma'}\frac{d \gamma'}{d t} = \frac{d \mathrm{R}}{d t}$$

$$\frac{d k_\varepsilon}{d a}\frac{d \varepsilon}{d t} + \frac{d k_\pi}{d a}\frac{d \pi}{d t} \ldots + \frac{d k_{\theta'}}{d a}\frac{d \theta'}{d t} = -\frac{d \mathrm{R}}{d a}$$

or taking for variables k_ε, k_π, &c., instead of $a, e, \gamma, a', e', \gamma'$, then the equations assume the canonical form,

$$\frac{d k_\varepsilon}{d t} = \frac{d \mathrm{R}}{d \varepsilon}, \ \frac{d k_\pi}{d t} = \frac{d \mathrm{R}}{d \pi}, \ \ldots$$

$$\frac{d \varepsilon}{d t} = -\frac{d \mathrm{R}}{d k_\varepsilon}, \ \frac{d \pi}{d t} = -\frac{d \mathrm{R}}{d k_\pi}, \ \ldots$$

The author remarks that the functions k_ε, k_π, k_θ, have much analogy with those which M. Delaunay has called L, G, H. Taking as variables l, g, h, instead of ε, π, θ, and forming the functions kl, kg, kh, in like manner with k_ε, k_π, k_θ, but attending only to the co-ordinates x, y, z, of the Moon, he obtains M. Delaunay's functions L, G, H, to a certain degree of approximation, but he has not attempted to demonstrate their rigorous identity.

The idea seems a very important one, and it may be anticipated that by thus starting from the problem of three bodies as if it were solved, and obtaining theorems in regard to determinate

functions (such as the above mentioned canonical elements) of the constants of the solution, some new light will be thrown on the original problem of three bodies. I have before remarked that in the ordinary investigations of Physical Astronomy, the problem treated of is not strictly that of three bodies, but a different and easier problem, that of disturbed elliptic motion.—ED.

Instrument for Sale.

Transit Instrument for Sale: aperture $2\frac{1}{4}$ inches, with 5 wires. A very perfect specimen of modern workmanship, made by Troughton and Simms for an eminent astronomical baronet now deceased, at a cost of about 50*l.*; complete in box with the usual fittings, iron stand, price 30*l.* Apply to G. F. Chambers, Bickley, Kent, where the Instrument may be seen.

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